Ecological Indicators 10 (2010) 427–441

Contents lists available at ScienceDirect

Ecological Indicators

journal homepage: www.elsevier.com/locate/ecolind

Urban permeation of landscapes and sprawl per capita: New measures of urban sprawl

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ARTICLE INFO

Article history:
Received 12 January 2009
Received in revised form 7 July 2009
Accepted 18 July 2009

Keywords:
Clumping
Dispersion
Landscape indices
Landscape structure
Scale
Spatial heterogeneity
Suitability criteria
Switzerland

ABSTRACT

Urban sprawl (dispersed urban development) has increased at alarming rates in Europe and North America over the last 50 years. Quantitative data are urgently needed in monitoring systems for sustainable development. However, there is a lack of reliable measures of urban sprawl that take into account the spatial configuration of the urban areas (not just total amount). This paper introduces four new measures of urban sprawl: degree of urban dispersion (*DIS*), total sprawl (*TS*), degree of urban permeation of the landscape (*UP*), and sprawl per capita (*SPC*). They characterize urban sprawl from a geometric point of view. The measures are related through $TS = DIS \times$ urban area, $UP = TS$/size of the landscape studied, and $SPC = TS$/number of inhabitants.

The paper investigates the properties of the new measures systematically using 13 suitability criteria which were derived from a clear definition of urban sprawl as discussed in a previous paper. The scale of analysis is specified by the so-called horizon of perception. Second, the new measures are applied to three examples from Switzerland. Subsequently, the measures are briefly compared to other measures of urban sprawl from the literature. We demonstrate that *UP* is an intensive and area-proportionately additive measure and is suitable for comparing urban sprawl among regions of differing size, while *SPC* is most appropriate when comparing sprawl in relation to human population density. The paper also provides practical advice for calculating the new measures. We conclude that the new method is more suitable than previous methods to quantify the indicator "urban sprawl" in monitoring systems as this method distinguishes the phenomenon of urban sprawl from its various causes and consequences. This article is part II of a set of two papers.

1. Introduction

The quantification of urban sprawl is a prerequisite for establishing verifiable objectives for environmental quality (e.g., limits to curtail urban sprawl), identification of trends and changes in trends (in time and space), for detecting statistical relationships between urban sprawl and ecological effects, and for an unambiguous determination of thresholds in the effects of urban sprawl. Therefore, quantitative information about the degree of urban sprawl is urgently needed to prepare a suitable indicator for monitoring systems on regional and national scales. In addition, it is useful for comparing landscape-management scenarios and to more effectively communicate scientific evidence to politicians and other decision-makers.

However, measures of urban sprawl that have been proposed in the literature (Ewing et al., 2003; Razin and Rosentraub, 2000; Wilson et al., 2003; Davis and Schaub, 2005; Tsai, 2005; Kasanko et al., 2006; Frenkel and Ashkenazi, 2008; Schneider and Woodcock, 2008; Torrens, 2008) report many differing dimensions. For example, Torrens (2008) distinguished eleven characteristics of urban sprawl and applied 42 different metrics to capture seven of these characteristics. While this approach provides researchers and planners with a wealth of information, not all of it needs to be reported in monitoring systems. Therefore, we advocate a more systematic approach based on suitability criteria (originally described in Jaeger et al., 2009) to focus on the core phenomenon by disentangling it from its causes and consequences. The 13 suitability criteria described by Jaeger et al. are: (1) intuitive interpretation, (2) mathematical simplicity, (3) modest data

DOI of original article: 10.1016/j.ecolind.2009.07.007
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doi:10.1016/j.ecolind.2009.07.010

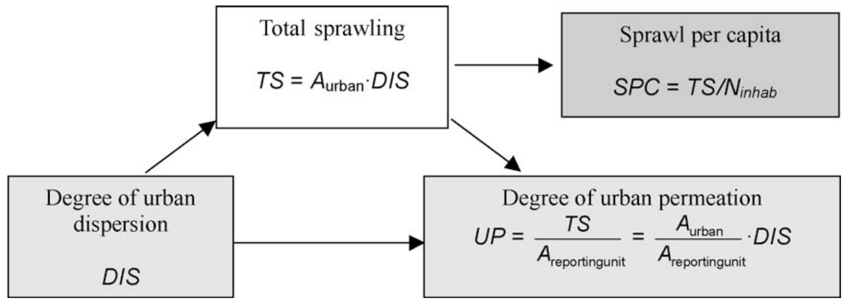

Fig. 1. The relationships between the four measures of urban sprawl *DIS*, *TS*, *UP*, and *SPC*. *DIS*, *UP*, and *SPC* are intensive measures (shaded), and *TS* is an extensive measure (white box). N_{inhab} = number of inhabitants of the reporting unit; A_{urban} = amount of urban area in the reporting unit, $A_{reporting\ unit}$ = size of the reporting unit.

requirements, (4) low sensitivity to very small patches of urban area, (5) monotonous response to increases in urban area, (6) monotonous response to increasing distance between two urban patches when within the scale of analysis, (7) monotonous response to increased spreading of three urban patches, (8) same direction of the metric's responses to the processes in criteria 5, 6 and 7, (9) continuous response to the merging of two urban patches, (10) independence of the metric from the location of the pattern of urban patches within the reporting unit, (11) continuous response to increasing distance between two urban patches when they move beyond the scale of analysis, (12) mathematical homogeneity (i.e., intensive or extensive measure), and (13) additivity (i.e., additive or area-proportionately additive measure) (see below Section 3). They are based on the following definition: "Urban sprawl is visually perceptible. A landscape suffers from urban sprawl if it is permeated by urban development or solitary buildings. The more urban area present in a landscape and the more dispersed the urban patches, the higher the degree of urban sprawl" (Jaeger et al.). The application of the criteria to three existing metrics (amount of urban area, proximity and contagion) in Jaeger et al. demonstrated that all three are severely limited in their suitability as a measure of urban sprawl. In fact, we are not aware of any existing measures of urban sprawl that meet all 13 criteria (see below Section 5).

Throughout the remainder of this paper, we use the following terminology: "urban patches" are patches of urban area, "urban points" (or "urban locations") are points located within urban area.

1.1. Objectives

The main objective of this paper is to introduce a reliable method to quantify the degree of urban sprawl. The method comprises a set of four related metrics called urban permeation, urban dispersion, total sprawl, and sprawl per capita (Fig. 1). We developed the new method in order to meet all 13 criteria described in Jaeger et al. To illustrate the new method, we used a set of examples from Switzerland, and briefly compared the new method with several existing methods.

2. Definition of the new measures of urban sprawl

2.1. Motivation and verbal definition

The new measures make use of the understanding that the degree of urban sprawl increases with both increasing amount of urban area and increasing dispersion. Accordingly, the new metrics characterize the pattern of urban areas in a geometric perspective and their calculation is based on all distances between any two points located within the urban areas, i.e., the contribution of each pair of urban points to the measure is based on the distance between the two points. The farther apart the two points, the higher their contribution. As a person placed at a particular location perceives the surrounding landscape (e.g., when hiking or seeking recreation) up to a certain maximum distance, there is an upper limit to the distances that are taken into account. Therefore, when the distances between two locations are larger than this maximum distance, urban development at the two locations is considered independently. We call this maximum distance (cutoff radius) the *horizon of perception* (*HP*). It represents the scale at which urban sprawl is investigated, e.g., at a more local scale (*HP* is low), or at a more regional scale (*HP* is high). This corresponds well with general insights from spatial analysis in that the degree of clumping or dispersion of some type of land cover can be studied on different scales. For example, a point pattern can exhibit clumping on a small scale while showing an evenly spaced distribution on a larger scale (e.g., Fortin and Dale, 2005). Such an upper limit of distances taken into account excludes situations where the contribution of one place (e.g., Zurich) to urban sprawl were influenced by urban development processes in places far away (e.g., Copenhagen), and prevents the sprawl measures from being dominated by contributions of large distances.

The *degree of urban dispersion* (*DIS*) is the average weighted distance between any two points chosen randomly within the urban areas in the landscape investigated, where the second point is chosen within a distance less than the horizon of perception (Fig. 2). A weighting of the distances is necessary to meet the suitability criteria (in particular criterion 7, see below Fig. 3). The weighting can be intuitively understood as describing the effort for delivering some service from one of the two points to the other, or for providing some kind of infrastructure between the two points

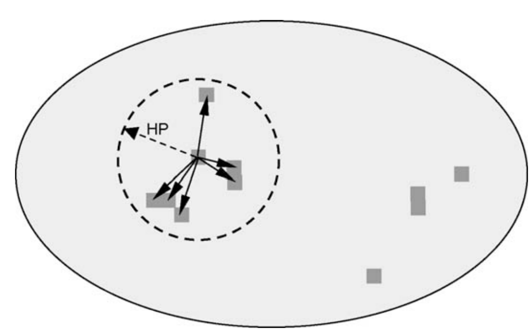

Fig. 2. Illustration of the new metrics and the horizon of perception (*HP*). The metrics are based on the distances between pairs of points within the urban area. The arrows indicate examples of distances between one location (in the center of the circle) and all other locations with a distance less than *HP*. The degree of urban dispersion (*DIS*) is the average "effort" required to connect from one randomly chosen point to another point within the horizon of perception of that first point (e.g., effort per m^2 of urban area). Total sprawl (*TS*) is the total average effort required to connect from every point to another point within the horizon of perception of the first point (see text). The weighting function that specifies the "effort" for connecting the points is illustrated in Fig. 3d.

(see below). The value of *DIS* does not depend on the total amount of urban area because the average effort of all pairs of points is considered (i.e., the expected value). Therefore, the farther away the newly built buildings from the existing ones, the higher the expected effort; and the more clumped the buildings (i.e., closer to each other), the lower the expected effort (Fig. 2). *DIS* is an *intensive* measure (criterion 12, see Jaeger et al. and Appendix C).

The second measure is called *total sprawl* (*TS*), and is defined as the average sum of the weighted distances between *all* points in the urban area and randomly chosen second points where each second point is not farther away from the first point than the horizon of perception. This measure can be intuitively understood as the expected total effort for delivering some service from all urban area (e.g., from every building) to randomly chosen delivery points within the starting point's "range of delivery" (=horizon of perception). The value of *TS* always increases when new areas are developed somewhere in the landscape investigated (see below for more details). It follows from this definition that *TS* is the product of *DIS* and the total amount of urban area (Fig. 1). It is an *extensive* measure (criterion 12).

The third measure is defined as *TS* divided by the total size of the reporting unit and is called *degree of urban permeation of the landscape* (*UP*; Fig. 1). It is an intensive measure. This measure can be intuitively understood as the average effort for delivering some service from all urban area (e.g., from all buildings) present within 1 km^2 of landscape on average, to randomly chosen delivery points within the horizon of perception of each starting point.

The fourth measure is called *sprawl per capita* (*SPC*) and is defined as *TS* divided by the number of inhabitants in the reporting unit. It is an intensive measure in relation to the number of inhabitants rather than the size of the reporting unit, and therefore can be compared among regions of differing size on a per capita basis. This metric establishes the relationship between the other three (purely geometric) sprawl metrics and population density (Fig. 1).

2.2. Derivation of the formulas

The metrics are applied to a binary map of the landscape that distinguishes built-up patches (urban patches) from open landscape. Then the formulas corresponding to the four measures defined above are as follows:

Degree of urban dispersion

$$= DIS = \frac{1}{A_{\text{urban}}} \int_{\vec{x}\in \text{urban areas}} \frac{1}{\int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} f(|\vec{x}-\vec{y}|)d\vec{y}d\vec{x}, \quad (1)$$

$$\text{Total sprawl} = TS = \int_{\vec{x}\in \text{urban areas}} \frac{1}{\int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} f(|\vec{x}-\vec{y}|)d\vec{y}d\vec{x}, \quad (2)$$

Urban permeation of the landscape

$$= UP = \frac{1}{A_{\text{reporting unit}}} \int_{\vec{x}\in \text{urban areas}} \frac{1}{\int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y}\in \text{urban areas and } |\vec{x}-\vec{y}|<HP} f(|\vec{x}-\vec{y}|)d\vec{y}d\vec{x}, \quad (3)$$

where A_{urban} is the total amount of urban area within the reporting unit, $A_{\text{reporting unit}}$ is the size of the reporting unit, *HP* is the horizon of perception, and $f(|\vec{x}-\vec{y}|)$ is the weighting function for the distances between any two points $\vec{x}$ and $\vec{y}$ including the unit (urban permeation units/m^2) = (UPU/m^2). The integrals for $\vec{x}$ and $\vec{y}$ are computed over the urban areas (with particular attention given to the boundaries, see below). The function $f(|\vec{x}-\vec{y}|)$ can assume different forms and is determined below.

The calculation of the three metrics can be easily based on a grid of cells (e.g., an ASCII-grid), where the integrals are approximated by sums over small cells of urban area, e.g., squares of width b:

$$DIS(b) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n_i}\left(\sum_{j=1}^{n_i} f(d_{ij}) + WCC(b)\right), \quad (4)$$

$$TS(b) = b^2 n \cdot DIS(b) = b^2 \cdot \sum_{i=1}^{n}\frac{1}{n_i}\left(\sum_{j=1}^{n_i} f(d_{ij}) + WCC(b)\right), \quad (5)$$

$$UP(b) = \frac{b^2 n}{A_{\text{reporting unit}}} \cdot DIS(b) = \frac{b^2}{A_{\text{reporting unit}}}\sum_{i=1}^{n}\frac{1}{n_i}\left(\sum_{j=1}^{n_i} f(d_{ij}) + WCC(b)\right), \quad (6)$$

where n_i denotes the number of urban cells that are closer to cell i than the horizon of perception, d_{ij} is the distance between (the centers of) cell i and cell j, and $WCC(b)$ is the within-cell contribution.

The within-cell contribution $WCC(b)$ of each urban cell needs to be included, i.e., in the case $j = i$ not just 0 is added (as suggested by the term in brackets as $d_{ii} = 0$) but the value of the cell itself. The larger the cell size, the more relevant the value of the within-cell contribution. Each within-cell contribution is rather small in comparison to the sum over j (in Eqs. (4)–(6)) but it will influence the result, in particular when there are only a few urban cells in the landscape. In practical terms, these values need to be calculated only once and then can be looked up in a table (Table 1).

Depending on how the boundaries of the reporting unit are treated (see below), n_i may include urban cells that are outside of the reporting unit but within the distance *HP* of cell i. The smaller the size of the cells, the better the approximation of the true values of *DIS*, *TS*, and *UP*:

$$\lim_{b\mapsto 0} DIS(b) = DIS, \quad \lim_{b\mapsto 0} TS(b) = TS, \quad \lim_{b\mapsto 0} UP(b) = UP. \quad (7)$$

2.3. Choice of a weighting function

The choice of the weighting function is based on the suitability criteria. A linear function, e.g., $f(|\vec{x}-\vec{y}|) = |\vec{x}-\vec{y}|$, does not meet criterion 7 (monotonous reaction to increased spreading of three urban patches) because it does not distinguish between a configuration of three urban patches in a row where two are clumped together and a configuration where they are distributed evenly, while the distance of the two patches at the ends is kept constant (Fig. 3a). In order to increase when the three patches are distributed more evenly, the weighting function has to increase more slowly than linearly. A convenient way to find a suitable function is to propose a sensible behaviour of such a function and solve the resulting differential equation. Firstly, the function should start at $f(0) = 0$; secondly, it should start with a slope of 1 at $x = 0$; and thirdly, when the distance between the two points is increased by a certain percentage, the value of the weighting function should increase by a certain proportion (constant factor) of that percentage, i.e., the increase Δf over f should be a certain proportion γ of the increase in the distance Δx over the distance x:

$$\frac{\Delta f(x)}{f(x)} = \gamma \frac{\Delta x}{x}. \tag{8}$$

This leads to the differential equation:

$$\frac{d f(x)}{dx} = \gamma \frac{f(x)}{x}, \tag{9}$$

which has the solution $f(x) = Cx^{\gamma}$ where C is a constant. The possible range of γ follows from the condition that an increase in x shall result in a higher value of f in the situation in Fig. 3a, i.e.:

$$\Delta f(x) + \Delta f(s-x) = \gamma \frac{\Delta x}{x} C x^{\gamma} - \gamma \frac{\Delta x}{s-x} C (s-x)^{\gamma} > 0. \tag{10}$$

It follows that $x^{\gamma-1} > (s-x)^{\gamma-1}$ when $s - x > x$, which implies $0 < \gamma < 1$.

The simplest of these functions is the square root function, i.e., $\gamma = 1/2$. However, as the slope of $f(x) = C\sqrt{x}$ is infinite at $x = 0$, we slightly modify this function to start at a point where the slope is 1. This can be achieved by shifting it down and to the left by a small amount and choosing $C = \sqrt{2}$. The resulting function is:

$$f(|\vec{x}-\vec{y}|) = \left(\sqrt{\frac{2|\vec{x}-\vec{y}|}{1\,\text{m}} + 1} - 1 \right) \frac{\text{UPU}}{\text{m}^2}, \tag{11}$$

where UPU denotes urban permeation units. In principle, other choices of $0 < \gamma < 1$ are possible, but $\gamma = 1/2$ is the simplest and most convenient value for the calculations.

Using this weighting function, the formulas of the three measures assume the form:

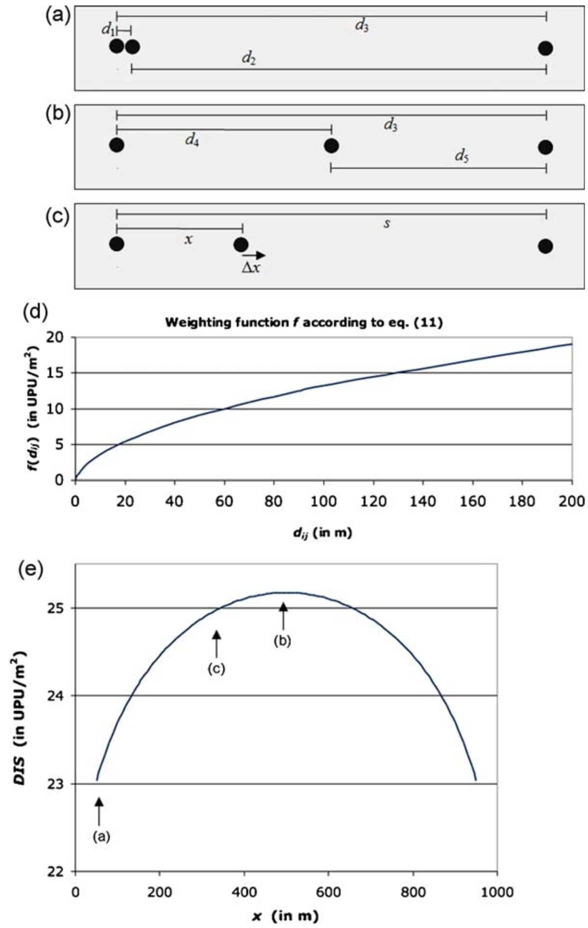

Fig. 3. Illustration of three patches of urban area in a linear configuration for deriving the weighting function. In configuration (a), the urban patches are more clumped than in (b). The sum of the distances is the same in both configurations ($d_1 + d_2 + d_3 = d_4 + d_5 + d_3$). Therefore, the weighting function has to increase less than linear to the distance. (c) When the urban patch in the centre is moved by Δx, the sum of the weighting functions for x and $s - x$ should increase, i.e., $\Delta f(x) + \Delta f(s-x) > 0$ (see text for details). (d) The weighting function $f(d_{ij})$ according to Eq. (11). (e) The resulting values of *DIS* (and *UP* accordingly) for the configurations shown in (a)–(c) (for d_1 = 50 m and d_3 = 1000 m). The value of *DIS* is lowest in (a) and highest in (b).

Even for relatively simple configurations of urban area, it is not possible to solve the integrals analytically. Therefore, some way of approximation or numerical calculation is needed (see Appendix A for a practical suggestion). In the approximation by a grid of cells as

$$DIS = \frac{1}{A_{\text{urban}}} \int_{\vec{x} \in \text{urban areas}} \frac{1}{\int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} \sqrt{\frac{2|\vec{x}-\vec{y}|}{1\,\text{m}} + 1} - 1 \; d\vec{y} d\vec{x} \frac{\text{UPU}}{\text{m}^2}, \tag{12}$$

$$TS = A_{\text{urban}} DIS = \int_{\vec{x} \in \text{urban areas}} \frac{1}{\int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} \sqrt{\frac{2|\vec{x}-\vec{y}|}{1\,\text{m}} + 1} - 1 \; d\vec{y} d\vec{x} \frac{\text{UPU}}{\text{m}^2}, \tag{13}$$

$$\begin{aligned} UP &= \frac{A_{\text{urban}}}{A_{\text{reporting unit}}} DIS \\ &= \frac{1}{A_{\text{reporting unit}}} \int_{\vec{x} \in \text{urban areas}} \frac{1}{\int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} d\vec{y}} \int_{\vec{y} \in \text{urban areas and } |\vec{x}-\vec{y}|<HP} \sqrt{\frac{2|\vec{x}-\vec{y}|}{1\,\text{m}} + 1} - 1 \; d\vec{y} d\vec{x} \frac{\text{UPU}}{\text{m}^2}. \end{aligned} \tag{14}$$

suggested above, the formulas are:

$$DIS(b) = \frac{1}{n}\sum_{i=1}^{n}\frac{1}{n_i}\left(\sum_{j=1}^{n_i}\left(\sqrt{\frac{2d_{ij}}{1\,\text{m}}+1}-1\right)\frac{\text{UPU}}{\text{m}^2}+WCC(b)\right), \qquad (15)$$

$$TS(b) = b^2\sum_{i=1}^{n}\frac{1}{n_i}\left(\sum_{j=1}^{n_i}\left(\sqrt{\frac{2d_{ij}}{1\,\text{m}}+1}-1\right)\frac{\text{UPU}}{\text{m}^2}+WCC(b)\right), \qquad (16)$$

$$UP(b) = \frac{b^2}{A_{\text{reporting unit}}}\sum_{i=1}^{n}\frac{1}{n_i} \times \left(\sum_{j=1}^{n_i}\left(\sqrt{\frac{2d_{ij}}{1\,\text{m}}+1}-1\right)\frac{\text{UPU}}{\text{m}^2}+WCC(b)\right). \qquad (17)$$

$WCC(b)$ is a function of cell size b and its value can be approximated for $0 < b < 1000$ m by the formula:

$$WCC(b) = \left(\sqrt{0.97428b/1\,\text{m}+1.046}-0.996249\right)\frac{\text{UPU}}{m^2}. \qquad (18)$$

Its values are given in Table 1. For example, if a landscape includes only one urban cell of width b, the value of DIS equals $WCC(b)$, e.g., 2.96 UPU/m^2 for b = 15 m and $TS(b) = b^2WCC(b)$ = 666.7 UPU.

Again, the smaller the size of the cells, the better the approximation of the true values of DIS, TS, and UP (Eq. (7)). The minimum dispersion for a given amount of urban area is assumed if they are clumped together in the shape of a circle (see Fig. 1 in Jaeger et al.). However, if the reporting unit is much larger than HP, the lowest value is assumed if each building is outside of the horizon of perception of all other buildings (which is only possible if the total amount of urban area is rather low).

Table 1
Values of the within-cell contribution $WCC(b)$ to the value of DIS (and the other metrics) used in Eqs. (4)–(6) as a function of cell width (b).

Cell width b (in m)	Within-cell contribution $WCC(b)$ (in UPU/m^2)
0	0
1	0.41853
2	0.73279
5	1.43842
10	2.29088
15	2.96326
20	3.53682
30	4.50733
45	5.70447
50	6.05853
60	6.71803
75	7.61312
90	8.42355
100	8.92714
150	11.13557
200	12.99981
300	16.13012
400	18.77086
500	21.09824
600	23.20286
700	25.13853
800	26.94043
900	28.63298
1000	30.2339

2.4. Treatment of the boundaries of reporting units

There are two options of how to treat the boundaries of reporting units:

1. *Cutting-out procedure*: Only the distances among urban points located within the reporting unit are taken into account, i.e., everything outside the boundary is neglected.
2. *Cross-boundary connections (CBC) procedure* (Moser et al., 2007): All distances between the urban points within the reporting unit and any other urban points that are smaller than the horizon of perception are taken into account regardless which reporting unit the surrounding urban points are located in, i.e., the second points include urban areas within a buffer zone around the reporting unit width of the horizon of perception (Fig. 4).

The cutting-out procedure has the advantage that no data are needed from areas outside of the reporting unit and that, as a consequence, the results are not influenced by urban development outside of the reporting unit. This corresponds to cutting the reporting unit out from its context. However, it has the disadvantage that the true context of the urban areas located close to the boundary is only partly considered even though these parts of the reporting unit will actually be influenced by all development processes surrounding them, including those on the other side of the boundary (Fig. 4). For example, a human being seeking recreation will perceive this location as suffering from urban sprawl if there are many developed areas visible, regardless of whether the buildings are located inside or outside of the reporting unit. In addition, the calculations for adjacent reporting units using the cutting-out procedure are not well related to the results for the combination of several adjacent reporting units because all the distances between urban areas located in reporting unit A and those in reporting unit B are neglected when calculated separately (but included when their combination is analyzed). The smaller the reporting units, the larger this bias.

The CBC procedure has the important advantage that all points within urban areas are treated equally regardless of how close they are to the boundary of some reporting unit. No distances between any two points of urban area that are smaller than HP are neglected. If they are across the boundary between two reporting units they are taken into account in the sprawl calculations of both reporting units (Fig. 4). This procedure solves the so-called

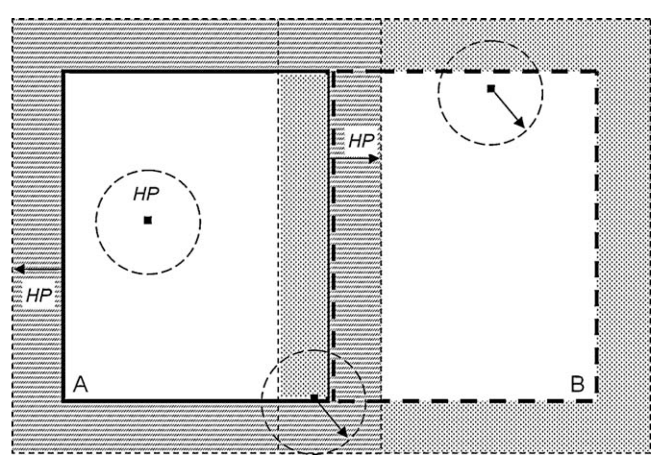

Fig. 4. Illustration of applying the cross-boundary connections procedure to determine urban permeation (UP). Shown are two very small urban patches in the reporting unit A and one very small urban patch in the reporting unit B. All distances between points within urban areas and other urban points located within the horizon of perception (HP) of the first point are taken into account, even when the other urban points are located in other reporting units. The buffers are of width HP to indicate the area around a reporting unit within which urban points may be included in the calculation of the value of UP.

boundary problem (Moser et al., 2007). It has been applied to other landscape metrics before, e.g., to the effective mesh size metric for quantifying the degree of landscape fragmentation (Moser et al., 2007). The only potential disadvantage of this treatment is that data outside of the reporting unit within a buffer width of *HP* need to be available.

As a consequence, the calculation of the measures according to the CBC procedure can be performed in a two-step procedure when an approximation based on raster cells is used. First, the values of *UP* for every cell of urban area can be calculated taking into account the distances to all other urban cells closer than *HP*. Second, the cells that are actually part of the reporting unit of interest are selected and their contributions to *UP* are added up. Their sum is divided by the size of the reporting unit, resulting in the value of *UP*.

Because of the advantages of the CBC procedure, this is the most appropriate method. In addition, it has the advantage that the metrics *UP* and *DIS* then are rigorously area-proportionately additive and *TS* is additive (criterion 13, see Appendix B). However, in some cases, the cutting-out procedure may also be useful, e.g., when the data about the areas outside of the reporting unit are not available.

3. Examining the suitability criteria

The new measures meet all 13 suitability criteria well or very well. Urban permeation (*UP*) meets the criteria directly (Table 2). *UP* is the main measure of urban sprawl proposed in this paper according to the definition presented above. The other three metrics are related to *UP* (Fig. 1) and the criteria apply accordingly (i.e., with a few modifications).

Total sprawl (*TS*) being the product of *UP* and the size of the reporting unit, is an extensive measure (criterion 12). It is also the product of the degree of dispersion and the total amount of urban area (rather than "...and the amount of urban area per unit area of the landscape"; criterion 1 in Table 2, item (3)). Consequently, *TS* is an additive measure (when the cross-boundary connections procedure is used; Fig. 4; criterion 13). Criteria 2–11 apply directly to *TS* without modification in the same way that they apply to *UP* (Table 2).

Sprawl per capita (*SPC*) is intensive in relation to population size, i.e., its value can be compared among regions with differing numbers of inhabitants (criterion 12). It is also the product of the degree of dispersion and the average urban area taken up by each inhabitant in the region investigated (rather than "...and the amount of urban area per unit area of the landscape"; criterion 1 in Table 2). As a consequence, *SPC* is population-proportionately additive, i.e., the value of *SPC* for the combination of two (or more) reporting units is the population-proportionate average of the values of the reporting units (when the cross-boundary connections procedure is used; criterion 13). Criteria 2–11 apply directly to *SPC* in the same way that they apply to *UP* without modification (Table 2).

The degree of dispersion (*DIS*) is the ratio of *UP* and the amount of urban area per km^2 and therefore is an intensive measure with regard to both landscape size and amount of urban area (criterion 12). Its value can be compared among regions of differing proportions of urban area. Criteria 2–3, 6–7, and 9–10 apply directly to *DIS* without modification in the same way as they apply to *UP* (Table 2). Criteria 1, 4 and 11 apply with slight (obvious) adjustments. As *DIS* can increase or decrease when new urban area is added to a landscape, criterion 5 does not apply to *DIS*. Consequently, criterion 8 does not apply to *DIS*, either. Criterion 13 applies to *DIS* in a modified form, e.g., the value of *DIS* for the combined reporting unit can be calculated via the value of *UP*.

4. Three examples from Switzerland

We applied the new metrics to three examples from Switzerland (Sursee, Chur, and Lugano; Fig. 5) as an illustration and to enhance the intuitive understanding of the metrics. Each example region is a circle of size 113.95 km^2, i.e., it has a diameter of 12,045 m (Fig. 5). The examples are based on the VECTOR25 data by *Swisstopo*, Berne, for 2002. Historic maps were digitized for 1960 and 1935. We compared the results for two horizons of perception (2 and 5 km). The settlement pattern outside the circles within the horizon of perception also influenced the values of the metrics through the cross-boundary connections procedure. Therefore, each characterization of the three regions includes a brief description of the surroundings of the circles.

The Sursee region is located in the Swiss Lowlands and is dominated by agriculture. The area includes many small villages and hamlets, and contains no larger towns. The settlements are embedded in the valleys of soft chains of hills running from the southeast to the northwest. The settlements are evenly distributed across the landscape, and this pattern is continued within 5 km around the circle. The second example is Chur which is located on an alluvial cone in a valley in the Alps with steep slopes. From there it grows into the valley bottom of the river Rhine which flows from the southwest to the northeast. A chain of a small number of villages follows the river, and this chain is continued outside the circle, but there the number of villages is rather low. The third example, Lugano, is located on a lake (to the southeast of the city). It is bordered by mountain ranges to the west and to the east. The development of settlements proceeded along the valley bottoms from the south to the north. To the north of the circle shown, the number of settlements is greatly reduced, and only a thin chain of villages continues. To the south, the settlement area is bordered by another lake, so there are almost no settlements outside the circle in this direction.

"Urban areas" used in these examples include residential and industrial areas. Only those traffic areas are included that are located within the settlements. Roads in the open landscape are not included because they do not contribute to "urban sprawl" according to our definition (see above) but constitute a different topic (i.e., landscape fragmentation, e.g., Jaeger et al., 2008; Girvetz et al., 2008). Some areas that are intensively used by humans, e.g., golf courses or outdoor sports facilities, are not included, either. However, the buildings located in such areas are taken into account.

With increasing horizon of perception, the values of the urban sprawl metrics also increase. Therefore, the values for the 5 km horizon of perception are always higher than those for the 2 km horizon of perception.

Both the amounts of urban area and their increases between 1935 and 2002 are very similar in Sursee and Chur (+111–113%), whereas Lugano has more urban area and a relative increase more than twice as high (+230%) (Table 3).

At all three times (1935, 1960, 2002), urban permeation was highest in the Lugano region and lowest in the Chur region (Fig. 6a). Between 1960 and 2002, *UP* has increased by more than three times as much as between 1935 and 1960 in all three regions. In general, *UP* increases more than urban area does, if *DIS* increases; if *UP* increases less than urban area, then *DIS* decreases.

For the 2 km horizon of perception, *DIS* is highest in Lugano. *DIS* has increased rather uniformly with increasing urban area in Lugano for both horizons of perception (Fig. 6b and c). There were already many small villages around the town of Lugano in 1935 which were at distances closer than 2 km to each other and therefore relevant for both horizons of perception (Fig. 5), and dispersion was already high. By 1960, new urban areas had been

Table 2
Examination of the new measure "Degree of urban permeation" (*UP*) with regard to the 13 suitability criteria for metrics of urban sprawl (+++ = very good, ++ = satisfying or good, + = slightly fulfilled, − = not fulfilled). The other three measures *DIS*, *TS* and *SPC* are closely related to *UP* (Fig. 1) and therefore, the suitability criteria apply accordingly, see text. For an assessment of the three measures amount of urban area, proximity, and contagion using the same criteria, see Table 2 in Jaeger et al.

Suitability criteria	Assessment of the measure "Degree of urban dispersion" (*UP*)	
	Suitability	Explanation
1. Intuitive interpretation	+++	The new metrics are based on the understanding of a landscape being the more sprawled, the more area is built-up and the more dispersed the buildings. Accordingly, the metric of urban permeation uses three intuitive ideas: (1) it describes the average effort of delivering some service from all urban points (e.g., every building) to randomly chosen delivery points (within a specified "range of delivery" representing the scale of analysis); (2) its value always increases when new urban areas are added; (3) it is the product of the degree of dispersion and the amount of urban area per unit area of the landscape. Therefore, *UP* is a direct expression of the definition of urban sprawl used in this paper.
2. Mathematical simplicity	++	*UP* is a second-order metric and its value is calculated as an integral over all pairs of points within the urban area of the landscape investigated. It does not depend on a particular cell size. Its value can be well approximated by a sum over all pairs of cells of urban area, and this approximation quickly converges towards the value of the integral (by reducing the size of the cells). Each pair of points (or cells) contributes to *UP* according to their distance and the respective value of the weighting function. The formula is conceptually straightforward and can be calculated numerically for any landscape and its pattern of urban area (see Appendix A) (it does not receive "+++" because a computer is required to calculate its value).
3. Modest data requirements	+++	The need for data is low. Maps of the areas classified as settlements (or "urban area") are sufficient. Usually, such maps are available in digital format (e.g., VECTOR25 1:25,000 by Swisstopo Berne). This is an ideal basis for calculating the new sprawl metrics using a Geographic Information System (GIS). To document historical states of urban development, the corresponding older maps need to be scanned and digitized.
4. Low sensitivity to very small patches of urban area	+++	The contribution of each patch of settlement area to *UP* is proportional to its size. Therefore, smaller and smaller patches have less and less influence on the metric's value (the calculation of the average effort for connecting two points is taken in relation to the size of patches; at no point is the number of patches used which would create the problem of disregarding patch size; Jaeger, 2000).
5. Monotonous reaction to increases in urban area	++	When new urban areas are added to a landscape, the value of *UP* always increases, except for a few rare exceptional cases of high dispersion where *UP* can be slightly reduced by building densely (see Appendix B). The amount by which *UP* increases will depend on the amount and the relative location of the new urban patches in relation to the existing pattern of urban areas. It is generally impossible to reduce the value of *UP* of a landscape by adding more urban area (except for some rare cases of building densely in a very dispersed situation).
6. Monotonous reaction to increasing distance between two urban patches when within the scale of analysis	+++	When the distance between two urban patches increases (while they are still within the horizon of perception of each other), the value of *UP* always increases. The increase of *UP* exhibits a shape that is similar to the shape of the effort function (Fig. 3).
7. Monotonous reaction to increased spreading of three urban patches	+++	This criterion is met through the choice of the weighting function increasing less than proportionally to the distance between points (Fig. 3). Therefore, the value of *UP* increases faster at shorter distances, i.e., the gain in *UP* due to increases in the distance to close urban patches is larger than the loss in *UP* due to decreases in the distance to distant urban patches (Fig. 3).
8. Same direction of the metric's responses to the processes in criteria 5, 6 and 7	+++	The responses of *UP* to the three processes referred to in criteria 5, 6 and 7 are all increasing, i.e., in the same direction.
9. Continuous reaction to the merging of two urban patches	+++	When two urban patches merge, the contribution of inter-patch distances to *UP* decreases continuously (i.e., from pairs of points each of which is located on a different patch). This is a consequence of the weighting function being a continuous function (including the point 0). The two intra-patch contributions do not change.
10. Independence of the metric from the location of the pattern of urban patches within the reporting unit	+++	The value of *UP* depends only on the spatial pattern of the urban area and on the size of the landscape investigated. Therefore, the value of *UP* is not changed when the entire pattern of urban area is rotated or moved to a different location within the landscape.
11. Continuous reaction to increasing distance between two urban patches when they move beyond the scale of analysis	+++	When the distance between two urban patches increases beyond the horizon of perception (that defines the scale of analysis) the value of *UP* changes continuously. For those parts of the inter-patch contribution that are based on pairs of points closer than the horizon of perception, their contribution increases, while the other parts do not contribute any more. *UP* changes continuously, because the decrease in the value of *UP* is proportional to the amount of urban area and the movement across the *HP* distance is a continuous process.
12. Mathematical homogeneity (i.e., intensive or extensive measure)	+++	*UP* is an intensive measure in relation to the size of the reporting unit (i.e., *UP* does not depend on the size of the reporting unit), and its value can be compared among reporting units of differing sizes.
13. Additivity (i.e., additive or area-proportionately additive measure)	+++	*UP* is an area-proportionately additive metric, i.e., the value of *UP* for the combination of two (or more) reporting units is the area-proportionate average of the values for the reporting units (when the cross-boundary connections procedure is used for the calculation of *UP*; Fig. 4), see proof in Appendix C.

added in the form of strands at the fringe of the main town and rather dispersed additions to the older villages. The new development by 2002 has extended the strands and has connected many of the surrounding villages forming elongated stripes. Therefore, dispersion has increased even further.

DIS increased even more steeply in Sursee and Chur between 1935 and 2002 than in Lugano for the 2 km horizon of perception. However, the value of *DIS* first decreased in Sursee between 1935 and 1960 (Fig. 6b). In 1935, the many villages in Sursee were mostly separated by distances larger than 2 km and therefore

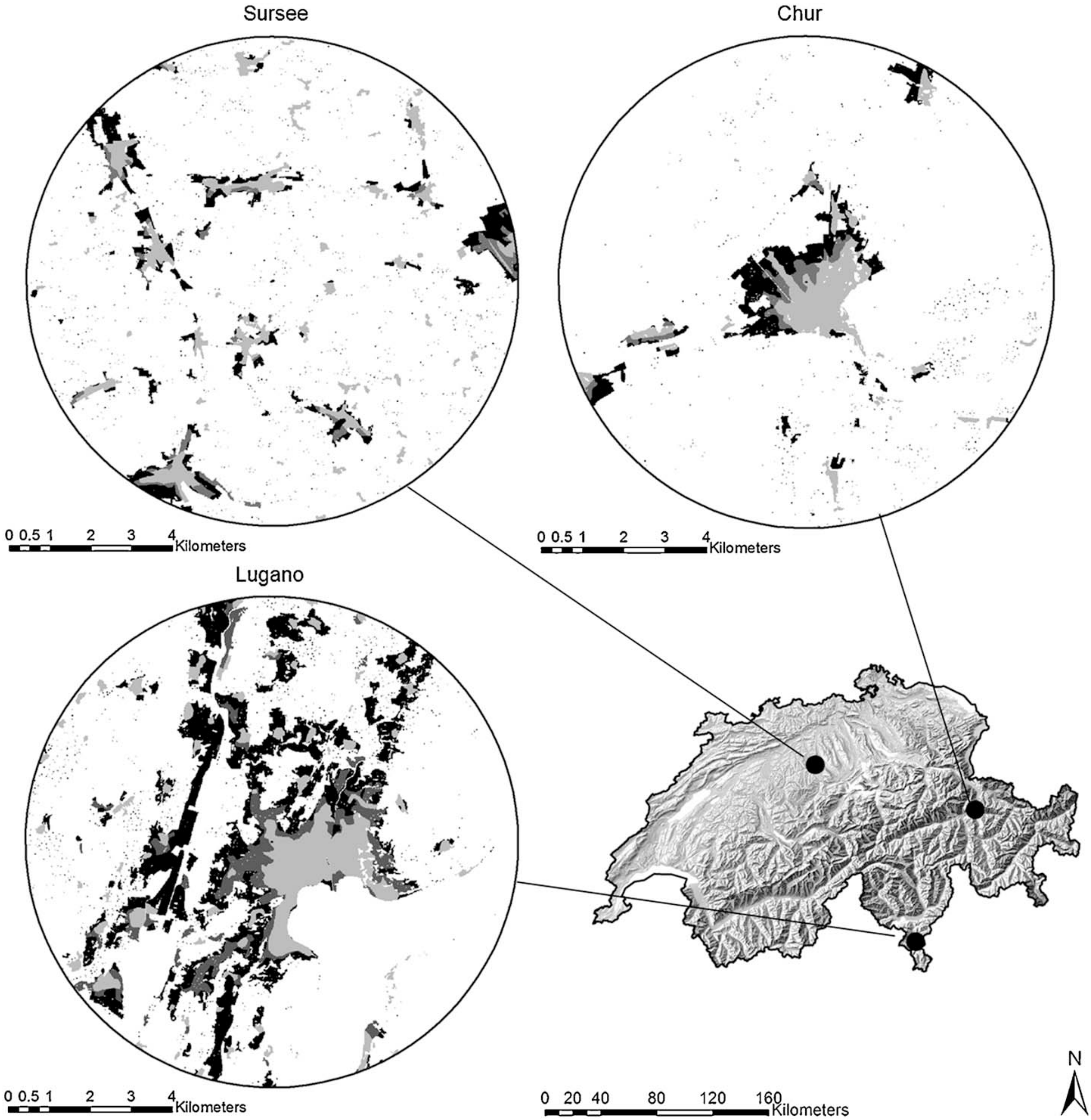

Fig. 5. Urban development in three regions from Switzerland used to illustrate the new urban sprawl metrics (Sursee, Chur, and Lugano). The diameter of each landscape is 12 km. The maps show the development of urban areas for three points in time: 1935 (in light grey), 1960 (in dark grey), and 2002 (in black).

contributed independently to the sprawl metrics for the 2 km horizon of perception. The urban areas that had been added by 1960 were located close to the existing villages and therefore were still not perceived from neighbouring villages (thus *DIS* decreased). Only after 1960 did the urban areas extend farther away from the villages and reduced the average distances between the boundaries of the villages to less than 2 km, which means that significant parts of neighbouring villages were now often within the horizon of perception of each village. Thus, *DIS* increased steeply between 1960 and 2002.

In Chur, the urban area was not broken up into as many independent small villages in 1935 at the 2 km scale as in Sursee; only about four small villages surround the main town and are far enough to be independent of it, i.e., >2 km (Fig. 5). Therefore, *DIS* is higher in Chur than in Sursee for the 2 km horizon of perception, whereas it is higher in Sursee than in Chur for the 5 km horizon of perception. This is clearly visible in the map of Sursee (Fig. 5) as each village includes in its 5 km horizon of perception three to five of its surrounding villages. This implies a much more scattered distribution of the urban areas at this scale than the distribution in

Table 3
Values of the three metrics urban dispersion (*DIS*), total sprawl (*TS*), urban permeation (*UP*) for two horizons of permeation (2 and 5 km), and the urban areas in the three example regions shown in Fig. 5 from Switzerland for three points in time (1935, 1960, 2002) (UPU = urban permeation units, MUPU = mega-UPU).

Region	Year	Urban area (ha)	Values of the sprawl metrics					
			Horizon of perception = 2 km			Horizon of perception = 5 km		
			DIS_2 (UPU/m^2)	TS_2 (MUPU)	UP_2 (UPU/km^2)	DIS_5 (UPU/m^2)	TS_5 (MUPU)	UP_5 (UPU/km^2)
Sursee	1935	532.5	41.64	221.73	1.946	76.50	407.31	3.574
	1960	671.1	41.38	277.74	2.437	75.06	503.76	4.421
	2002	1126.1	43.52	490.05	4.300	73.89	832.13	7.302
Chur	1935	443.4	42.28	187.46	1.645	61.68	503.76	2.400
	1960	550.8	42.75	235.46	2.066	60.98	335.85	2.947
	2002	946.6	45.06	426.59	3.743	64.61	611.64	5.367
Lugano	1935	858.8	46.13	396.19	3.477	69.53	597.08	5.240
	1960	1358.1	47.08	639.36	5.611	70.94	963.40	8.454
	2002	2862.5	47.82	1368.97	12.013	74.79	2140.96	18.788

the concentrated arrangement of the town of Chur where the town is surrounded by only one or two small villages (the third at the northeast border of the region is almost independent for the 5 km horizon). This difference also explains why *DIS* continues to decrease in Sursee between 1960 and 2002 for the 5 km horizon of perception. At this scale, the new urban areas fill in the space between the villages in a rather dense form, i.e., denser than the distribution of the villages in 1935 (*TS* behaves the same way as *UP* because all regions are of same size).

The broken lines indicate the value of *DIS* for an even distribution of urban cells width of 15 m (i.e., maximum value of *DIS*) and for a configuration as a circle. The area of a circle with diameter of 2 km is 313.2 ha, and 1963.5 ha for 5 km; therefore, the lower curves end at these values. For *HP* = 2 km, up to four circles of 2 km diameter can fit into the 113.95 km^2 landscape with distances >2 km, and the corresponding four lines are included in Fig. 6b.

The three examples illustrate very clearly that it is important to keep in mind what the horizon of perception is when interpreting the values of the metrics.

5. Discussion

5.1. Utility of the new metrics

For the interpretation of the results for a particular region, the values of *UP*, *DIS* and *SPC* should be compared. *UP* describes to what degree a landscape is permeated by settlement areas and solitary buildings. *SPC* relates sprawl to the number of inhabitants. As industrial areas often have low numbers of inhabitants, *SPC* can also be defined in relation to the number of jobs in a region (or to the sum of inhabitants and jobs).

When new buildings are added within the existing urban patches (densification), then the values of *UP* and *DIS* do not change, whereas *SPC* decreases according to the number of new inhabitants and/or jobs. This corresponds well with the intuitive understanding that urban areas of higher densities are less sprawled. Therefore, it may be convenient to identify "sprawl" as a particular combination of certain ranges of values of *UP*, *DIS* and *SPC*. This may include attempts to quantitatively define "sprawl" based on the combined values of *UP*, *DIS* and *SPC*. For example, such a quantitative definition could exclude city centers from the term "sprawl" when *SPC* is higher than a certain threshold even though *UP* is high (and *DIS* intermediate). Such ranges will be suggested based on empirical data from Switzerland in a separate paper.

The values of the metrics will differ depending on the definition of "urban area", e.g., whether or not solitary buildings or areas taken up by transportation infrastructure in the open landscape are included. Therefore, a reliable definition and delineation of urban areas is a prerequisite for the quantification of *UP* and *DIS*. Tools such as VectorGen can be used to objectively delineate urban areas (Millward, 2002, 2004).

Attempts to standardize *DIS* to range between 0 and 1 should be treated with caution for several reasons. Standardization would likely affect the relationship $UP = DIS \times A_{\text{urban}}$. This relationship is one of the main advantages of the new metrics introduced in this paper (Fig. 1) and should be maintained. Standardization of *DIS* might also compromize the validity of criterion 5 for *UP*. In addition, the convenient property of *UP* being intensive and area-proportionately additive (criteria 12 and 13) should not be put at risk. Instead of trying to change the role of *DIS* within its relationship with *UP*, the value of *DIS* can always be reported separately and interpreted in comparison with *UP* and *SPC*.

One major advantage of the new metrics over many other landscape metrics is that their definitions and values do not depend on any cell sizes. The cells used in Eqs. (4)–(6) and (15)–(17) serve to approximate the integrals given in Eqs. (1)–(3) and (12)–(14), and all cell sizes produce the same results (when the cells are not too large, e.g., less than 50 m). This is a consequence of the benign convergence behaviour of the approximations. We performed a test of how well the approximations converge towards the true values (see Appendix A) and found that:

1. the values of *DIS*(b) converge with the refinement of the approximation (i.e., increasing number of cells) towards the values that were calculated with *Mathematica*;
2. the inclusion of the within-cell contribution *WCC*(b) leads to increased convergence of the approximation;
3. the inclusion of the within-cell contribution *WCC*(b) in the approximations leads to more accurate results, in particular when the cell size is rather large. The influence of the within-cell contribution on accuracy of the results decreases when cell size decreases.

Based on these tests and on our experience from applying the new sprawl metrics to Switzerland, we recommend the choice of b = 15 m.

The contributions of those urban patches located closer to the boundary of the reporting unit than the horizon of perception are influenced by urban areas outside of the reporting unit if these patches are within the horizon of perception (Fig. 4). This is achieved through the application of the CBC procedure which removes any bias that would otherwise be produced by ignoring the context of a reporting unit, e.g., by the cutting-out procedure (Moser et al., 2007).

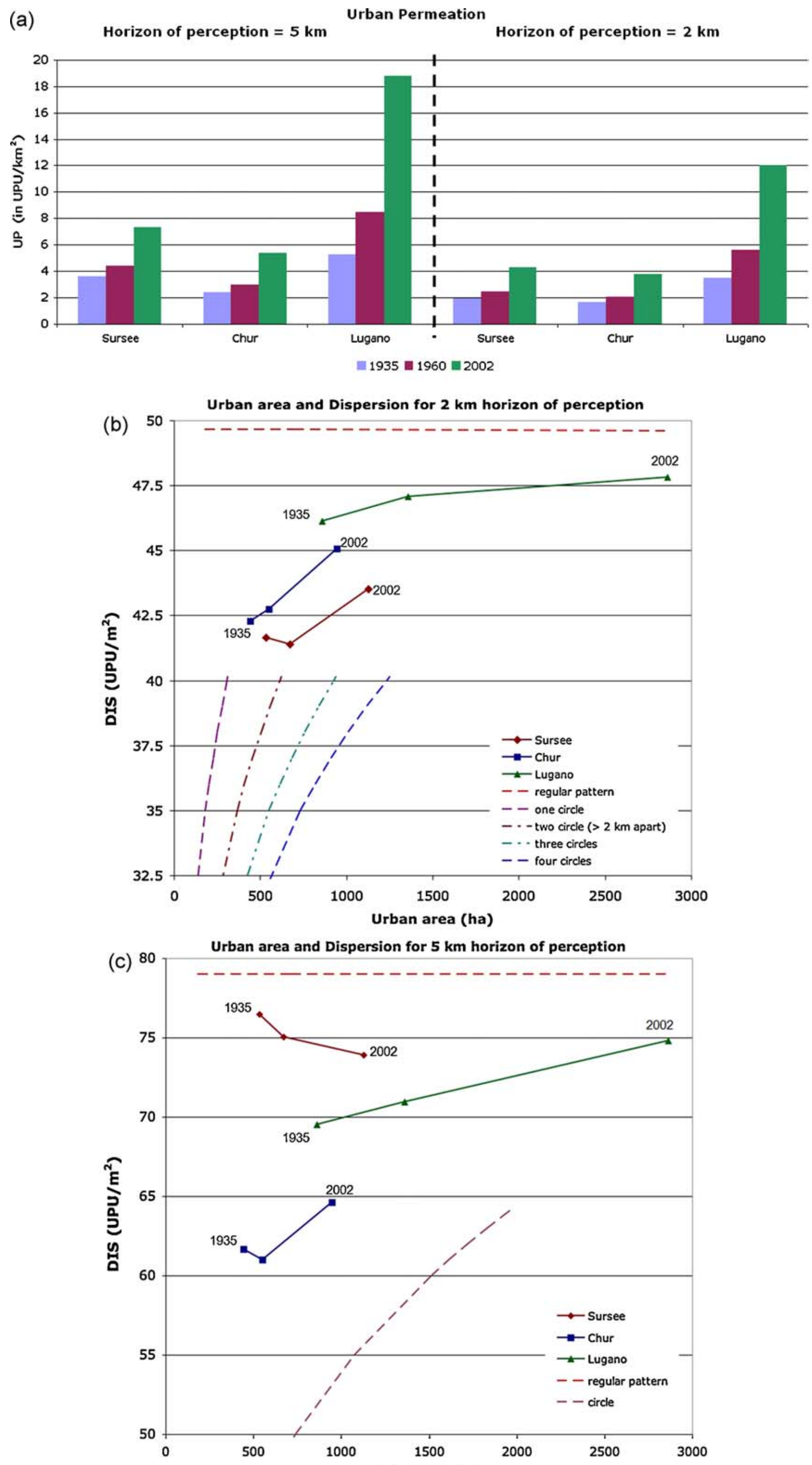

Fig. 6. Development of urban permeation (*UP*), urban dispersion (*DIS*) and urban area in the three example regions shown in Fig. 5 between 1935 and 2002 for two horizons of permeation (2 and 5 km). For comparison, the values of *DIS* for a regular distribution of 15 m × 15 m cells and for a solid circle (up to four circles for *HP* = 2 km) of urban area are indicated by broken lines (in b and c), see text. The data points in the center are values for 1960.

5.2. Suitability criteria for measures of urban sprawl

This paper applies the 13 suitability criteria to ensure that the new metrics meet all requirements for measures of urban sprawl. The application of suitability criteria provides a reliable approach to understanding the behaviour of landscape metrics in a systematic way (Jaeger, 2000; Jaeger et al.) and is useful for preventing misunderstanding and misuse of landscape metrics which is a common issue in landscape ecology (Li and Wu, 2004).

5.3. Brief comparison with other measures of urban sprawl

Jaeger et al. assessed three metrics in detail: amount of urban area, proximity, and contagion. None of them meets all 13 criteria.

The amount of urban area by itself, though an important component of urban sprawl and widely used, does not include information about the spatial arrangement of urban areas and therefore is not sufficient to measure urban sprawl.

Methods from spatial analysis that are frequently used to assess whether point patterns are random, clumped, or regular include the *K* function and quadrat tests of randomness (Cressie, 1993; Bailey and Gatrell, 1995; Fotheringham et al., 2000; Fortin and Dale, 2005). However, these methods do not apply to continuous spatial patterns. They are based on counts of point events, rather than continuous areas (which cannot be counted in a point-wise manner). However, the new metrics are to some degree related to the *K* function (Bailey and Gatrell, 1995), in the sense that urban locations at a certain distance around each urban point are considered, but here they are weighted (not just counted).

An important advantage of the new metrics is that they include both intra-patch distances and inter-patch distances. This is a major reason why the merger of two (or more) urban patches does not produce a "jump" in the values of the metrics (criterion 9). Such "jumps" constitute a major drawback of the proximity metric introduced by Whitcomb et al. (1981; see also Gustafson and Parker, 1992, 1994). In addition, the proximity metric does not meet the direction criterion (criterion 8 in Jaeger et al.).

Various other measures that have been suggested to quantify certain aspects of urban sprawl have been discussed in Jaeger et al., e.g., (1) percentage of dwellings in single-unit detached houses, (2) population per square kilometer, and (3) housing units per square kilometer (Razin and Rosentraub, 2000). These earlier measures do not, to our knowledge, meet the 13 suitability criteria. In addition, most of them do not explicitly account for the need to analyze urban sprawl on differing scales, e.g., contagion (see Jaeger et al.). A detailed comparison of other existing measures and the new metrics to substantiate these claims will be performed in a separate paper.

The new measures are second-order metrics. Both first-order and second-order metrics are meaningful for quantifying landscape patterns. Most landscape metrics calculate first-order statistics, e.g., patch area, road density, patch shape metrics (McGarigal and Marks, 1995). First-order statistics describe the variation in the intensity of some process at individual locations (or events), whereas second-order characteristics summarize point-to-point relationships (Wiegand and Moloney, 2004). In general, second-order properties describe the spatial dependence between events at any two locations, i.e., they "examine the correlations or covariances between events occurring in two distinct points or regions" (Fotheringham et al., 2000: 140). To measure the spatial configuration of urban areas, the distances to all other points within urban area are relevant (if they are located within the horizon of perception). Several other landscape metrics have been proposed in the literature that have second-order properties. These include the ecologically scaled landscape index average patch connectivity (Vos et al., 2001), which is the probability that a patch is colonized based on species-specific movement distances and the spatial configuration of habitat patches. Other examples are the effective mesh size (Jaeger, 2000; Girvetz et al., 2008), Ripley's *K* function and the O-ring statistic (Wiegand and Moloney, 2004).

6. Conclusions

To measure urban sprawl, the spatial arrangement of the urban areas needs to be taken into account. The method for quantifying urban sprawl introduced in this paper meets all 13 suitability criteria for measures of urban sprawl and has produced convincing results for Switzerland (Wissen et al., submitted for publication). The four new metrics can be used separately to characterize urban sprawl, or in combination to identify urban sprawl as a specific association of certain value ranges of the four metrics.

The new metrics are useful to measure the speed of urban development, identify trends (e.g., densification or increasing dispersion), compare urban sprawl among different regions, and to suggest quantitative limits to curtail urban sprawl. The properties of the new metrics are particularly convenient for the comparison of regions of differing size because they are intensive measures (and even area-proportionately additive measures).

The four new metrics have recently been applied to Switzerland (on a time series since 1935) in two projects that are part of the National Research Programme 54 "Sustainability of the Built Environment" by the Swiss National Science Foundation (Wissen et al., submitted for publication). The results are planned to be used as an indicator in the Swiss Monitoring System of Sustainable Development (MONET; SFSO et al., 2004) and in the Swiss Spatial Monitoring Program (run by the Swiss Federal Office for Spatial Development ARE and the Swiss Federal Office for the Environment FOEN).

Urban sprawl can be measured on different scales. Therefore, the four new metrics include a parameter called "horizon of perception" (*HP*) that specifies the scale of analysis. As illustrated by the three examples from Switzerland, the scale of analysis is important to consider in the interpretation of the results. Recommendations for the choice of the horizon of perception can be based on the following estimation: due to the curvature of the earth, the distance of perception for a human being (with eye-height of 1.80 m) is $a = 4.9$ km on a surface with no obstacles (calculated by using the Pythagorean formula $a^2 + (6370\text{ km})^2 = (6370\text{ km} + 1.80\text{ m})^2$, where 6370 km is the average radius of the earth). Therefore, distances between 1 and 10 km seem most suitable. The horizon of perception may also be chosen in accordance with the type of urban development investigated and with the historical settlement structures. For example, if new urban development reduces the distances between the boundaries of neighbouring towns or villages and this process is considered relevant for assessing urban sprawl, then the horizon of perception should be chosen larger than this distance. Based on our experience from applying the new metrics to Switzerland, we recommend choosing a value for *HP* of 2 and 5 km for regions with rather small-scale settlement structures such as Switzerland. To investigate at what scales the relevant sprawl processes are taking place, we recommend to use several *HP*s in parallel and to compare the results.

A computer program for automated calculation of the metrics is available from the authors.

Acknowledgements

We thank Michael Wenzlaff and Beat Trachsler for their generous programming support. We also thank Stefan Keller for his fruitful collaboration in software development. This work is part of a project within the Swiss National Research Programme (NRP 54) "Sustainable Development of the Built Environment" which was funded by the Swiss National Science Foundation (NSF); we particularly thank Stefan Husi from the NSF for supporting the collaboration among the authors of this paper. We thank John Lowry and Hugh Millward for inspiring discussions about measuring urban sprawl and two reviewers for helpful comments on the manuscript.

Appendix A. On the numerical calculation of the metrics

It is convenient to first calculate the value of S_i for each cell i in the landscape that the reporting units of interest are embedded in:

$$S_i = \frac{1}{n_i}\left(\sum_{k=1}^{n_i}\left(\sqrt{\frac{2 \cdot d_{ik}}{1\text{ m}} + 1} - 1\right) + WCC(b)\right), \tag{A1}$$

where n_i is the number of urban cells within the *HP* of cell *i*. For any chosen reporting unit, the three metrics can then be calculated based on the S_i values of the cells located within the reporting unit:

$$DIS(b) = \frac{1}{n}\sum_{i=1}^{n} S_i \frac{\text{UPU}}{\text{m}^2}, \tag{A2}$$

where *n* is the number of urban cells in the reporting unit,

$$TS(b) = b^2 \cdot \sum_{i=1}^{n} S_i \frac{\text{UPU}}{\text{m}^2}, \tag{A3}$$

where *b* is the width of the cell (in m),

$$UP(b) = \frac{b^2}{A_{\text{reporting unit}}} \sum_{i=1}^{n} S_i \frac{\text{UPU}}{\text{m}^2} = \frac{1}{A_{\text{reporting unit}}} TS(b). \tag{A4}$$

The approximation of the metrics based on cells converges quickly. Even cell sizes of 50 m × 50 m provide good results (Fig. 7).

In order to speed up the calculation of the metrics for large reporting units (e.g., large countries), the urban areas can be represented by cells that are only partially filled with urban development. This implies that the cells can be larger than the smallest patch of urban development taken account of in the calculations, e.g., larger than the size of solitary buildings in the landscape (>15 m). The calculation is faster because it includes fewer cells. The price paid for this advantage is that the accuracy in the calculation of the distances is lower. The degree of "urbanization" of a cell can be represented by values between 0 and 100% indicating the percentage of area of the cell covered by development. The formulas for *UP*, *DIS*, and *TS* will then need to be modified accordingly to include these percentage values.

Appendix B. Examination of *UP* with regard to suitability criterion 5

Because of the horizon of perception, the behavior of *UP* is in some cases non-trivial. When new urban areas are added to a landscape, the value of *UP* always increases, except for a few rare exceptional cases where *UP* can be slightly reduced by building densely in a very dispersed situation. (This effect disappears for different choices of *HP*.)

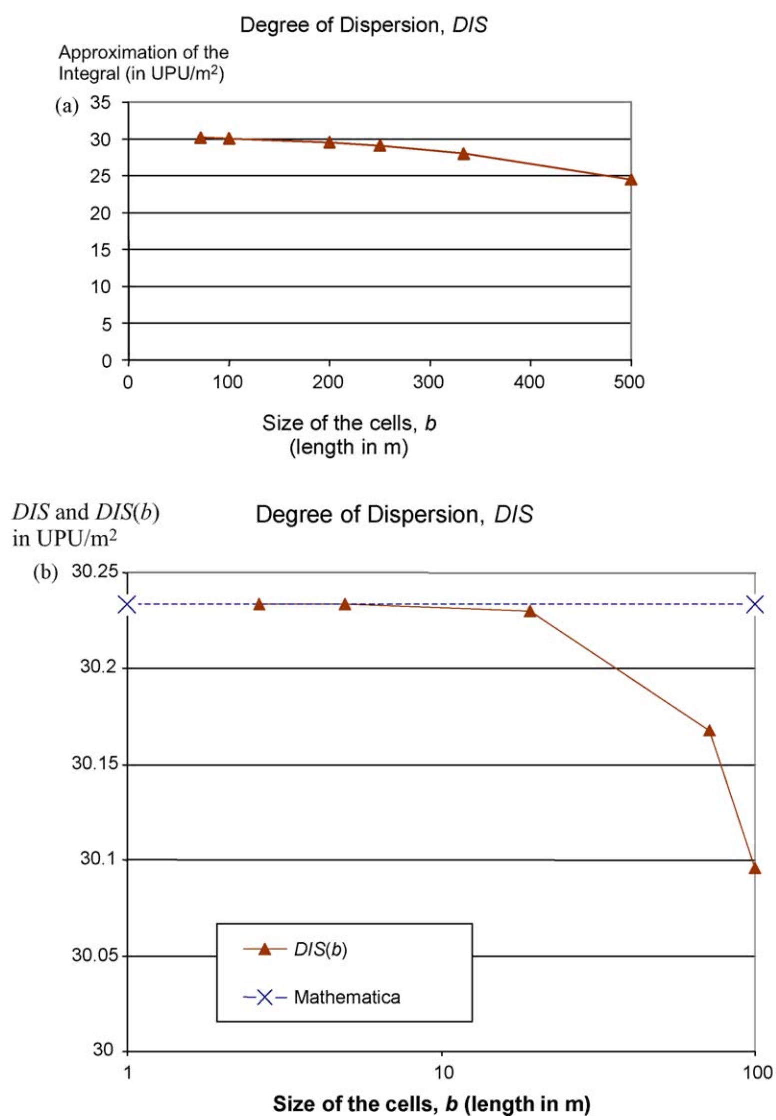

Fig. 7. Calculation of the degree of dispersion, *DIS*, for a square patch of urban area size of 1 km^2 through approximation of the integral Eq. (12) by the formula based on cells of varying size Eq. (15). (a) overall picture for cell sizes smaller than 500 m, and (b) logarithmic diagram for cell sizes smaller than 100 m. The approximation approaches the true value of 30.2339 UPU/m^2 (numerical calculation using *Mathematica*, see Table 1) very quickly when the size of the cells is smaller than 50 m. (The horizon of perception does not influence these results as long as it is larger than the largest distance between urban cells within the 1000 m* 1000 m square, i.e., $HP > \sqrt{2}$ km = 1.4142 km).

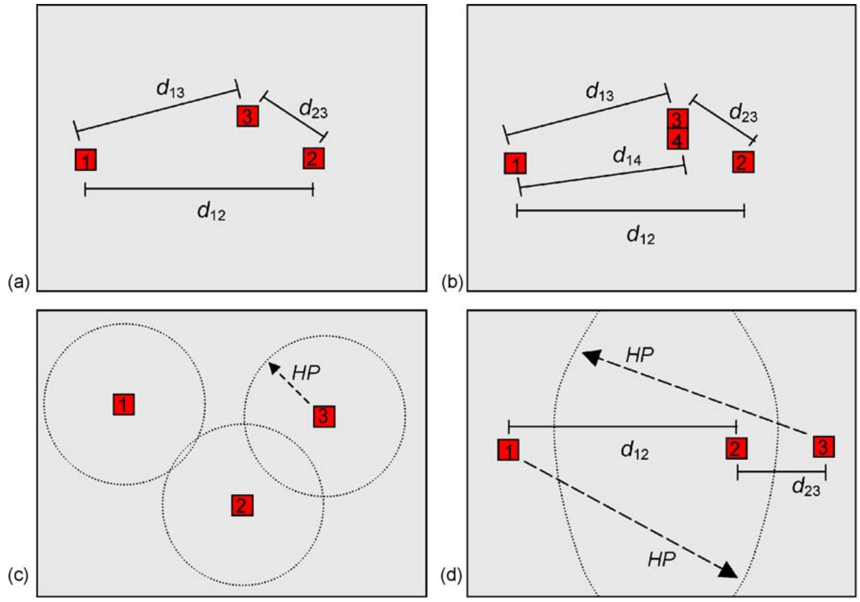

Fig. 8. Illustration of the four cases used for studying the response of *UP* to increases in urban area (criterion 5). In configuration (a), patch 3 was added and *UP* increased (*HP* was larger than the largest distance between any two urban patches, see text). (b) When another patch was added (patch 4), *UP* increased again, see text (d_{24} and d_{34} are not shown to avoid cluttering). (c) When patch 3 was added in a situation where the patches were outside of each other's *HP*, then *UP* always increased. (d) Only in a situation when the new patch (patch 3) is outside of the *HP* of patch 1, then *UP* can increase or decrease, depending on the distances between patches 2 and 3 and between patches 1 and 2, see text.

Proof: Four cases are distinguished (Fig. 8; all patches shown are of width *b*, i.e., urban cells, with no loss of generality):

In configuration (a), patch 3 was added and *UP* increased: Before adding patch 3, *UP* was

$$UP_{12}=\frac{b^2}{A_{ru}}\sum_{i=1}^{2}S_i=\frac{b^2}{A_{ru}}\left(\frac{1}{2}(f(d_{12})+WCC_b)+\frac{1}{2}(f(d_{12})+WCC_b)\right),$$

where A_{ru} is the area of the reporting unit. After adding patch 3, *UP* is

$$UP_{123}=\frac{b^2}{A_{ru}}\left(\frac{1}{3}(f(d_{12})+f(d_{13})+WCC_b+f(d_{12})+f(d_{23})+WCC_b+f(d_{13})+f(d_{23})+WCC_b)\right)$$
$$=\frac{b^2}{A_{ru}}\left(WCC_b+\frac{2}{3}f(d_{12})+\frac{2}{3}f(d_{23})+\frac{2}{3}f(d_{13})\right)$$
$$>\frac{b^2}{A_{ru}}\left(WCC_b+\frac{4}{3}f(d_{12})\right)>UP_{12},$$

using $f(d_{13})+f(d_{23})>f(d_{12})$. This holds true wherever patch 3 is located, as long as it is within the *HP* of patch 1 and patch 2.

(b) When another patch was added (patch 4), *UP* continued to increase: After adding patch 4, *UP* is

$$UP_{1234}=\frac{b^2}{A_{ru}}\left(WCC_b+\frac{2}{4}(f(d_{12})+f(d_{13})+f(d_{23})+f(d_{14})+f(d_{24})+f(d_{34}))\right)$$
$$>\frac{b^2}{A_{ru}}\left(WCC_b+\frac{2}{4}\left(\frac{3}{2}f(d_{12})+\frac{3}{2}f(d_{13})+\frac{3}{2}f(d_{23})\right)\right)$$
$$=\frac{b^2}{A_{ru}}\left(WCC_b+\frac{3}{4}(f(d_{12})+f(d_{13})+f(d_{23}))\right)>UP_{123},$$

using $\frac{1}{2}(f(d_{14})+f(d_{24}))>\frac{1}{2}f(d_{12})$, etc.

This can be continued for any number of urban cells.

(c) When patch 3 is added in a situation where the patches are outside of each other's *HP*, then *UP* always increases:

$$UP_{(c)}=\frac{b^2}{A_{ru}}(WCC_b+WCC_b+WCC_b)=3\cdot\frac{b^2}{A_{ru}}WCC_b,$$

which is simply the sum of the contributions of each patch.

(d) Only in a situation when the new patch (patch 3) is outside of the *HP* of patch 1, then *UP* can increase or decrease, depending on the distance between patches 2 and 3.

$$UP_{12}=\frac{b^2}{A_{ru}}\left(\frac{1}{2}(f(d_{12})+WCC_b)\cdot 2\right)$$

and

$$UP_{123} = \frac{b^2}{A_{\text{ru}}}\left(\frac{1}{2}(f(d_{12}) + WCC_b) + \frac{1}{3}(f(d_{12}) + f(d_{23}) + WCC_b) + \frac{1}{2}(f(d_{23}) + WCC_b)\right).$$

Thus,

$$\Delta UP = UP_{123} - UP_{12} = \frac{b^2}{A_{\text{ru}}}\left(\frac{1}{3}WCC_b + \frac{5}{6}f(d_{23}) - \frac{1}{6}f(d_{12})\right).$$

When d_{23} is similar to d_{12} then ΔUP clearly is positive. However, if d_{23} is much smaller than d_{12}, this term can become negative. For example, when d_{12} = 9900 m, HP = 10,000 m, d_{23} = 200 m, then

$$\Delta UP = \frac{b^2}{A_{\text{ru}}}\left(15.854\tfrac{\text{UPU}}{\text{m}^2} + 0.988\tfrac{\text{UPU}}{\text{m}^2} - 23.29\tfrac{\text{UPU}}{\text{m}^2}\right) = -1450.8\frac{\text{UPU}}{A_{\text{ru}}} < 0,$$

using WCC_b from Table 1.

Appendix C. On the mathematical property of *UP* to be area-proportionately additive

Definitions

A landscape metric, say *F*, is called **'intensive'**, if $F(\lambda \cdot \Phi) = F(\Phi)$ for all configurations of urban area Φ and all $\lambda \in N$ where $\lambda \cdot \Phi$ is defined as the multiplication of the region represented by Φ in the same spatial arrangement of urban patches (cf. Chandler, 1987, pp. 22–25; Legendre and Legendre, 1998, p. 31). For example, for $\Phi = \{1\text{ha}, 4\text{ha}, 5\text{ha}\}$ a multiplication by $\lambda = 2$ results in $2\Phi = \{1\text{ha}, 1\text{ha}, 4\text{ha}, 4\text{ha}, 5\text{ha}, 5\text{ha}\}$, etc.

A landscape metric, say *F*, is called '**area-proportionately additive**' if the value of *F* for the combination of two urban area configurations Φ_1 and Φ_2 (with total areas $A^{(1)}_{\text{total}}$ and $A^{(2)}_{\text{total}}$) is given by

$$F(\Phi_1 \cup \Phi_2) = \frac{A^{(1)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \cdot F(\Phi_1) + \frac{A^{(2)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \cdot F(\Phi_2).$$

This is analogous to the way that temperature or the concentration of a liquid is determined: when two liquids are mixed, the concentration of the mixture becomes

$$c = \frac{V_1}{V_1 + V_2}c_1 + \frac{V_2}{V_1 + V_2}c_2$$

with V_j and c_j denoting the volumes and concentrations. This means that each part (e.g., Φ_1 and Φ_2) contributes proportionally to its size, even if each part has a different spatial structure. The characteristics of being intensive or area-proportionately additive are interrelated. 'Area-proportionately additive' means more than 'intensive'. In fact, every area-proportionately additive quantity is intensive. The reverse generally does not hold. Average patch size is an example of an intensive measure that is not area-proportionately additive.

Proof that UP is area-proportionately additive

Urban permeation, when calculated according to the CBC procedure, is an area-proportionately additive quantity (without any restrictions).

Proof: Let Φ_1 and Φ_2 be two configurations of urban area $\Phi_1 = \left\{A^{(1)}_i \middle| i = 1, \ldots, n_1\right\}$, $\Phi_2 = \left\{A^{(2)}_i \middle| i = 1, \ldots, n_2\right\}$ with total areas $A^{(1)}_{\text{total}}$ and $A^{(2)}_{\text{total}}$ of the two reporting units. Calculate the values of S_i for all cells in the two reporting units based on some cell size (b) according to formula (A1) given above. For any reporting unit, *UP* is then given by formula (A4).

Therefore, the value of *UP* for the joint configuration $(\Phi_1 \cup \Phi_2)$ results in

$$\begin{aligned} UP(\Phi_1 \cup \Phi_2) &= \frac{b^2}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}}\left(\sum_{i=1}^{n_1} S_i \frac{\text{UPU}}{\text{m}^2} + \sum_{j=1}^{n_2} S_i \frac{\text{UPU}}{\text{m}^2}\right) \\ &= \frac{A^{(1)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \frac{1}{A^{(1)}_{\text{total}}} \sum_{i=1}^{n_1} S_i \frac{\text{UPU}}{\text{m}^2} + \frac{A^{(2)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \frac{1}{A^{(2)}_{\text{total}}} \sum_{i=1}^{n_1} S_i \frac{\text{UPU}}{\text{m}^2}, \\ &= \frac{A^{(1)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \cdot UP(\Phi_1) + \frac{A^{(2)}_{\text{total}}}{A^{(1)}_{\text{total}} + A^{(2)}_{\text{total}}} \cdot UP(\Phi_2) \end{aligned}$$

where n_1 is the number of urban cells in reporting unit 1 und n_2 is the number of urban cells in reporting unit 2. This means that *UP* is an area-proportionately additive quantity. (Note that this equation does not hold true when the cutting-out procedure is used, because then the S_i-values would be different for cells close to the boundary that separates the two reporting units, see Fig. 4, as the connections between urban points on either side of the boundary would be missing.)

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